

# Anomalous Hall effect in ferromagnetic semiconductors

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(February 1, 2008)

We present a theory of the anomalous Hall effect in ferromagnetic (Mn,III)V semiconductors. Our theory relates the anomalous Hall conductance of a homogeneous ferromagnet to the Berry phase acquired by a quasiparticle wavefunction upon traversing closed paths on the spin-split Fermi surface of a ferromagnetic state. It can be applied equally well to any itinerant electron ferromagnet. The quantitative agreement between our theory and experimental data in both (In,Mn)As and (Ga,Mn)As systems suggests that this disorder independent contribution to the anomalous Hall conductivity dominates in diluted magnetic semiconductors.

In recent years the semiconductor research community has enjoyed a remarkable achievement, making III-V compounds ferromagnetic by doping them with magnetic elements. The 1992 discovery<sup>1</sup> of hole-mediated ferromagnetic order in (In,Mn)As has motivated research on GaAs<sup>2</sup> and other III-V host materials. Ferromagnetic transition temperatures in excess of 100 Kelvin<sup>3,4</sup> and long spin-coherence times in GaAs<sup>5,6</sup> have fueled hopes that a new magnetic medium is emerging that could open radically new pathways for information processing and storage technologies. The recent confirmation<sup>7</sup> of the room temperature ferromagnetism predicted<sup>8</sup> in (Ga,Mn)N has added to interest in this class of materials. In both (In,Mn)As and (Ga,Mn)As systems, measurements of the anomalous Hall effect<sup>9–14</sup> have played a key role in establishing ferromagnetism, and in providing evidence for the essential role of hole-mediated coupling between Mn local moments in establishing long-range order.<sup>1,2,15</sup> Despite the importance of the anomalous Hall effect (AHE) for sample characterization, a theory which allows these experiments to be interpreted quantitatively has not been available. In this article we present a theory of the AHE in ferromagnetic III-V semiconductors that appears to account for existing observations.

The Hall resistivity of ferromagnets has an ordinary contribution, proportional to the external magnetic field strength, and an anomalous contribution usually assumed to be proportional to the sample magnetization. The classical theory of the anomalous Hall effect (AHE) in a metal<sup>12</sup> starts from the mean-field Stoner theory description of its ferromagnetic state, in which current is carried by quasiparticles in spontaneously spin-split Bloch bands. A similar mean-field theory has recently been developed<sup>16–19</sup> and used to interpret magnetic properties of (III,Mn)V ferromagnets. In these theories the host semiconductor valence bands are split by an effective field that results from exchange interactions with polarized Mn moments. The field makes a wavevector independent contribution,

$$H_{split} = h \hat{m} \cdot \vec{s} \quad (1)$$

to the band Hamiltonian. Here  $\hat{m}$  is the polarization direction of the local moments and  $\vec{s}$  is the electron spin-operator. In the (In,Mn)As and (Ga,Mn)As AHE measurements,<sup>1,2</sup>  $\hat{m}$  is in the  $\langle 00\bar{1} \rangle$  direction for positive external magnetic fields. The effective field  $h$  is proportional to the average local moment magnetization and is non-zero only in the ferromagnetic state. The antiferromagnetic interaction<sup>20,21</sup> between localized and itinerant spins implies that  $h > 0$ . When Mn spins are fully polarized,  $h = N_{Mn} S J_{pd}$ , where  $N_{Mn}$  is the density of Mn ions with spin  $S = 5/2$  and  $J_{pd} = 50 \pm 5$  meVnm<sup>3</sup> is the strength of the exchange coupling between the local moments and the valence band electrons.<sup>15</sup> From a symmetry point of view, the AHE is made possible by this effective magnetic field, and by the spin-orbit coupling present in the host semiconductor valence band.

In the standard model of the AHE in metals, skew-scattering<sup>9</sup> and side-jump<sup>11</sup> scattering give rise to contributions to the Hall resistivity proportional to the diagonal resistivity  $\rho$  and  $\rho^2$  respectively, with the latter process tending to dominate in alloys because  $\rho$  is larger. Our evaluation of the AHE in (III,Mn)V ferromagnets is based on a theory of semiclassical wave-packet dynamics, developed previously by one of us<sup>22</sup> which implies a contribution to the Hall conductivity independent of the kinetic equation scattering term. Our focus on this contribution is motivated in part by practical considerations, since our current understanding of (III,Mn)V ferromagnets is not sufficient to permit confident modeling of quasiparticle scattering. The relation of our approach to standard theory is reminiscent of disagreements between Smit<sup>9</sup> and Luttinger<sup>10</sup> that arose early in the development of AHE theory and do not appear to have ever been fully resolved. In this paper we follow Luttinger<sup>10</sup> in taking the view that there is a contribution to the AHE due to the change in wavepacket group velocity that occurs when an electric field is applied to a ferromagnet. Since the Hall resistivity is invariably smaller than the

diagonal resistivity, a temperature independent value of the Hall conductivity corresponds to a Hall resistivity proportional to  $\rho^2$ , usually interpreted as evidence for dominant side-jump scattering. As we explained below, we find quantitative agreement between our Hall conductance values and experiment, suggesting that the AHE *conductance* value may be intrinsic in many metallic ferromagnets.

The Bloch electron group velocity correction is conveniently evaluated using expressions derived by Sundaram and Niu<sup>22</sup>:

$$\dot{x}_c = \frac{\partial \epsilon}{\hbar \partial \vec{k}} + (e/\hbar) \vec{E} \times \vec{\Omega}. \quad (2)$$

The first term on the right-hand-side of Eq. 2 is the standard Bloch band group velocity. Our anomalous Hall conductivity is due to the second term, proportional to the  $\vec{k}$ -space Berry curvature  $\vec{\Omega}$ . It follows from symmetry considerations that for a cubic semiconductor under lattice-matching strains and with  $\hat{m}$  aligned by external fields along the  $\langle 001 \rangle$  growth direction, only  $\Omega_z \neq 0$ :

$$\Omega_z(n, \vec{k}) = 2\text{Im}[\langle \frac{\partial u_n}{\partial k_y} | \frac{\partial u_n}{\partial k_x} \rangle]. \quad (3)$$

Here  $|u_n\rangle$  is the periodic part of the  $n$ -th Bloch band wavefunction with the mean-field spin-splitting term included in the Hamiltonian. The anomalous Hall conductivity that results from this velocity correction is

$$\sigma_{AH} = -\frac{e^2}{\hbar} \sum_n \int \frac{d\vec{k}}{(2\pi)^3} f_{n,\vec{k}} \Omega_z(n, \vec{k}), \quad (4)$$

where  $f_{n,\vec{k}}$  is the equilibrium Fermi occupation factor for the band quasiparticles. We have taken the convention that a positive  $\sigma_{AH}$  means that the anomalous Hall current is in the same direction as the normal Hall current.

This Berry phase contribution occurs for any itinerant electron ferromagnet. To assess its importance for (III,Mn)V compounds, we first explore a simplified model that yields parabolic dispersion for both heavy-hole and light-hole bands and a spin-orbit coupling<sup>23,18</sup> strength that is much larger than the hole Fermi energy. Detailed numerical calculations that account for mixing of the spin-orbit split-off bands and warping of the occupied heavy-hole and light-hole bands<sup>23,18</sup> in (In,Mn)As and (Ga,Mn)As samples<sup>1,3</sup> will follow this general and qualitative discussion. Within a 4-band model, the spin operator  $\vec{s} = \vec{j}/3$  in Eq.(1), and the spherical model Hamiltonian for holes in III-V host semiconductors can be written as

$$H_0 = \frac{\hbar^2}{2m} [(\gamma_1 + \frac{5}{2}\gamma_2)k^2 - 2\gamma_2(\vec{k} \cdot \vec{j})^2] \text{Ry} a_0^2, \quad (5)$$

where  $\vec{j}$  is the total angular momentum operator,  $\gamma_1$  and  $\gamma_2$  are Luttinger parameters,<sup>23,24</sup> and  $a_0$  is the Bohr radius. In the unpolarized case ( $h = 0$ ), the total Hamiltonian,  $H = H_0 + H_{split}$ , is diagonalized by spinors

$|j_{\vec{k}}\rangle$  where, e.g.,  $j_{\vec{k}} \equiv \vec{j} \cdot \hat{k} = \pm 3/2$  for the two degenerate heavy-hole bands with effective mass  $m_{hh} = m/(\gamma_1 - 2\gamma_2)$ . The Berry phase is familiar in this case since the Bloch eigenstates are  $j = 3/2$  spin coherent states<sup>25</sup>. Integrating over planes of occupied states at fixed  $k_z$  we find that  $\int d^2k f_{n,\vec{k}} \Omega_z(n, \vec{k}) = \pm 3/2(\cos\theta_{\vec{k}} - 1)$  where  $\cos(\theta_{\vec{k}}) \equiv k_z/k_{hh}$  and  $k_{hh}$  is the Fermi wavevector. The anomalous Hall conductivity (4) vanishes in the  $h = 0$  limit because the contributions from the two heavy hole bands, and also from the two light hole bands, cancel. In the ferromagnetic state, on the other hand, majority and minority spin heavy and light hole Fermi surfaces differ and also the Berry phases are modified when  $h \neq 0$ . Up to linear order in  $h$  we obtain that  $k_{hh}^{\pm} = k_{hh} \pm \cos\theta_{\vec{k}} h m_{hh}/(2\hbar^2 k_{hh})$  and the Berry phase is altered by the factor  $(1 \mp 2mh/(9\gamma_2 \hbar^2 k_{hh}^2))$ . A similar analysis for the light-hole bands leads to a total net contribution to the AHE from the four bands whose lower and upper bounds are:

$$\begin{aligned} \frac{e^2}{2\pi\hbar} \frac{h}{2\pi\hbar^2} (3\pi^2 p)^{-1/3} m_{hh} &< \sigma_{AH} < \\ \frac{e^2}{2\pi\hbar} \frac{h}{2\pi\hbar^2} (3\pi^2 p)^{-1/3} 2^{2/3} m_{hh}. \end{aligned} \quad (6)$$

Here  $p = k_{hh}^3/3\pi^2 (1 + \sqrt{m_{lh}/m_{hh}})$  is the total hole density and  $m_{lh} = m/(\gamma_1 + 2\gamma_2)$  is the light-hole effective mass. The lower bound in Eq.(6) is obtained assuming  $m_{lh} \ll m_{hh}$  while the upper bound is reached when  $m_{lh} \approx m_{hh}$ .

Based on the above analysis we conclude that the Berry phase anomalous velocity can yield a sizeable AHE in (III,Mn)V ferromagnets. The solid line in Fig.1 shows our analytic results for GaAs effective masses  $m_{hh} = 0.5m$  and  $m_{lh} = 0.08m$ . Note that in experiment, anomalous Hall conductances are in order of  $1\text{--}10 \text{ } \Omega^{-1} \text{cm}^{-1}$  and the effective exchange field  $h \sim 10\text{--}100 \text{ meV}$ . According to Eq. (6) larger  $\sigma_{AH}$  values should be expected in systems with larger heavy-hole effective masses and in systems with the ratio  $m_{lh}/m_{hh}$  close to unity.

So far we have discussed the limit of infinitely strong spin-orbit coupling with an exchange field that is small relative to the hole Fermi energy. In the opposite limits of zero spin-orbit coupling or large  $h$ ,  $\sigma_{AH}$  vanishes. This implies that the anomalous Hall conductivity is generally nonlinear in the exchange field and the magnetization. To explore the intermediate regime we numerically diagonalized the 6-band Luttinger Hamiltonian<sup>23,18</sup> with the spin-orbit gap  $\Delta_{so} = 1 \text{ eV}$  as well as for the GaAs value  $\Delta_{so} = 341 \text{ meV}$ . The results shown in Fig.1 confirm that smaller  $\sigma_{AH}$  is expected in systems with smaller  $\Delta_{so}$  and suggest that both positive and negative signs of  $\sigma_{AH}$  can occur in general. The curves in Fig.1 are obtained by neglecting band warping in III-V semiconductor compounds. The property that the valence bands in these materials are strongly non-parabolic, even in the absence of the field  $h$  and even in the large  $\Delta_{so}$  limit, is accurately

captured by introducing the third phenomenological Luttinger parameter  $\gamma_3$ .<sup>23,18</sup> Our numerical results indicate that warping leads to an increase of  $\sigma_{AH}$ , as seen when comparing the solid curves in Fig.1 and in the top panel of Fig.2. The hole-density dependence of  $\sigma_{AH}$ , illustrated in Fig.2, is qualitatively consistent with the spherical model prediction (6). Also, in accord with the chemical trends outlined above, the numerical data in Fig.2 suggest large positive AHE coefficient for (Al,Mn)As, an intermediate positive  $\sigma_{AH}$  in (Ga,Mn)As, and a relatively weak AHE in (In,Mn)As with a sign that may be sensitive to strain and other details of a particular sample.

We now compare our  $\sigma_{AH}$  theory with the experimental data available in (In,Mn)As and (Ga,Mn)As samples, studied extensively by Ohno and coworkers.<sup>1,3,4,15</sup> The nominal Mn densities in these two systems are  $N_{Mn} = 0.23 \text{ nm}^{-3}$  for the InAs host and  $N_{Mn} = 1.1 \text{ nm}^{-3}$  for the GaAs host, yielding saturation values of the effective field  $h = 25 \pm 3 \text{ meV}$  and  $h = 122 \pm 14 \text{ meV}$ , respectively. The low-temperature hole density of the (Ga,Mn)As sample,  $p = 0.35 \text{ nm}^{-3}$ , was unambiguously determined<sup>15</sup> from the ordinary Hall coefficient measured at high magnetic fields. Since similar experiments have not been reported for the (In,Mn)As sample, we estimated the hole density,  $p = 0.1 \text{ nm}^{-3}$ , by fitting the density-dependent mean-field theory  $T_c$  to the measured value  $T_c = 7.5 \text{ K}$ . The use of a mean-field theory description of the ferromagnetic state in both samples is justified<sup>26,27</sup> by the homogeneity of the samples and by the relatively small Fermi energy density of states. Indeed, the measured ferromagnetic transition temperature for the (Ga,Mn)As sample,  $T_c = 110 \text{ K}$ , is in an excellent agreement with the calculated transition temperature<sup>17,28</sup>, and mean-field theory also successfully explains the magnetic anisotropy of both systems.<sup>17,18</sup> Luttinger parameters for the two host semiconductors are well known<sup>24</sup> and are listed in the caption of Fig.2. As demonstrated in Fig.2, our theory explains the order of magnitude difference between AHE's in the two materials ( $\sigma_{AH} \approx 1 \Omega^{-1} \text{ cm}^{-1}$  in (In,Mn)As and  $\sigma_{AH} \approx 14 \Omega^{-1} \text{ cm}^{-1}$  in (Ga,Mn)As). The calculations are also consistent with the positive sign and monotonic dependences of  $\sigma_{AH}$  on sample magnetizations.<sup>15</sup>

We take the agreement in both magnitude and sign of the AHE as a strong indication that the anomalous velocity contribution dominates AHE in homogeneous (III,Mn)V ferromagnets. This Berry phase conductivity, which is independent of quasiparticle scatterers, is relatively easily evaluated with high accuracy. According to our theory, comparison of theoretical and experimental Hall conductivity values provides information not only on the magnetization but also on the character of the itinerant electron wavefunctions that participate in the magnetism. For example, we predict that size quantization effects in quantum wells that inhibit heavy-light hole mixing will reduce the  $\vec{k}$ -space Berry curvatures and hence anomalous Hall conductivities. The success reported here motivates a reexamination of this effect in

all itinerant electron ferromagnets.

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## ACKNOWLEDGMENTS

We are grateful for helpful discussions with T. Dietl, J. Furdyna, F. Matsukura, and H. Ohno. Our work was supported by DARPA, the Indiana 21st Century Fund, the Welch Foundation, the Ministry of Education of the Czech Republic, and the Grant Agency of the Czech Republic.

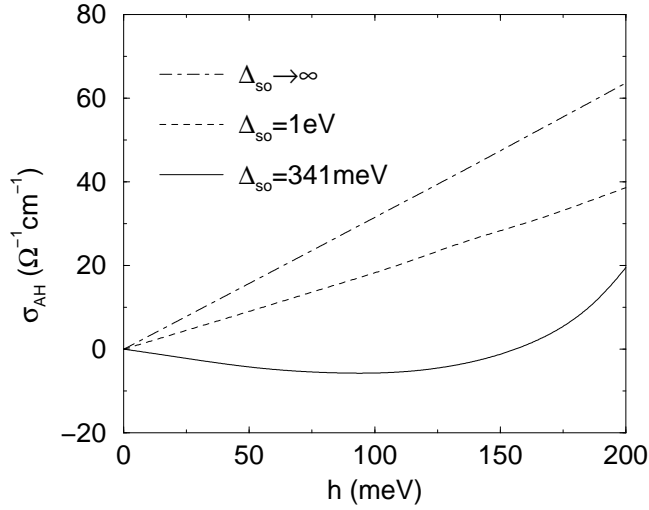


FIG. 1. Illustrative calculations of the anomalous Hall conductance as a function of polarized Mn ions field for hole density  $p = 0.35 \text{ nm}^{-1}$ . The dotted-dashed curve was obtained assuming infinitely large spin-orbit coupling and the decrease of theoretical  $\sigma_{AH}$  with decreasing spin-orbit coupling strength is demonstrated for  $\Delta_{so} = 1 \text{ eV}$  (dashed line) and  $\Delta_{so} = 341 \text{ meV}$  (solid line).

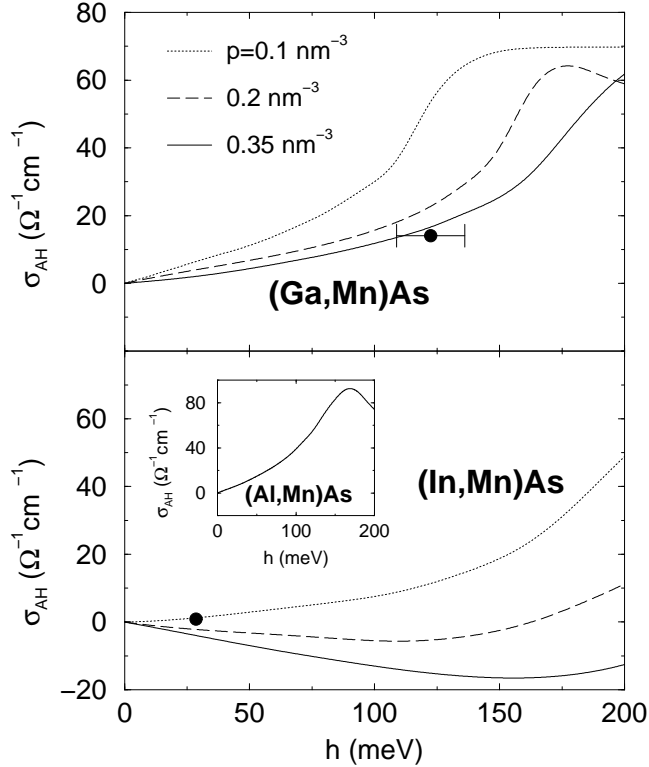


FIG. 2. Full numerical simulations of  $\sigma_{AH}$  for GaAs host (top panel), InAs host (bottom panel), and AlAs host (inset) with hole densities  $p = 0.1 \text{ nm}^{-1}$  (dotted lines),  $p = 0.2 \text{ nm}^{-1}$  (dashed lines), and  $p = 0.35 \text{ nm}^{-1}$  (solid lines). Following Luttinger parameters of the valence bands were used:<sup>24</sup> GaAs –  $\gamma_1 = 6.98$ ,  $\gamma_2 = 2.06$ ,  $\gamma_3 = 2.93$ ,  $\Delta_{so} = 341 \text{ meV}$ ; InAs –  $\gamma_1 = 20$ ,  $\gamma_2 = 8.5$ ,  $\gamma_3 = 9.2$ ,  $\Delta_{so} = 390 \text{ meV}$ ; AlAs –  $\gamma_1 = 3.76$ ,  $\gamma_2 = 0.82$ ,  $\gamma_3 = 1.42$ ,  $\Delta_{so} = 280 \text{ meV}$ . Filled circles in the top and bottom panels represent measured AHE.<sup>1,3,15</sup> The saturation mean-field  $h$  values for the two points were estimated from nominal sample parameters.<sup>1,3,15</sup> Horizontal error bars correspond to the experimental uncertainty of the  $J_{pd}$  coupling constant. Experimental hole density in the (Ga,Mn)As sample is  $p = 0.35 \text{ nm}^{-1}$ ; for (In,Mn)As,  $p = 0.1 \text{ nm}^{-1}$  was determined indirectly from sample's transition temperature.